Building Real Subspaces for Projection Based Model Order Reduction with Application in Computational Electromagnetics

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Model order reduction techniques have been used in the last decades to perform fast simulations of sets of equations obtained by field computation methods like PEEC. More specifically, moment matching approaches have been especially successful for this application. However, some attention must be paid in computation of the projection subspace to be able to produce real state space equations for the reduced system. In order to do this, a modified Arnoldi iteration is presented in this paper. This technique assures that the computed projection matrix is real and orthogonal. A loop antenna, obtained by PEEC, is used as an application example with very encouraging results.

Index Terms **— Dynamical Systems, Model Order Reduction, Moment Matching and PEEC.**

I. Introduction

The advances in the past decades have allowed the simulation of electromagnetic systems to a high degree of accuracy. The Finite Elements Method (FEM) and the Partial Element Equivalent Circuit (PEEC) are among of the most successful techniques [1]. These methods can produce High-Fidelity Models (HFM) for a variety of applications. However, if the problem is too complex or the required accuracy is very high, many differential equations are obtained.

It is in this context that Model Order Reduction (MOR) techniques can be applied to obtain fast and accurate solutions. Instead of directly solving the equations of the HFM, one can construct a smaller model that represent very well the input/output behavior of the original system but is faster to compute. There are some well-established methods to achieve this like Balanced Truncations, Proper Orthogonal Decomposition and Moment Matching [2]. For the reduction of electrical circuits, like those obtained by PEEC techniques, PRIM and SPRIM are particularly successful [3].

One way of expressing the differential equations obtained in numerical modeling is shown in (1). In this equation A and $E \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times p}$, $C \in \mathbb{R}^{q\times n}$, $x \in \mathbb{R}^{n}$, $u \in \mathbb{R}^{p}$ and $y \in \mathbb{R}^{q}$. The number n is the order of the model and p and q are, respectively, the number of inputs and outputs of the system. The vector x is the state vector of the dynamical system and u and y are its input and output vectors, respectively.

$$\begin{cases} E\dot{x} &= Ax + Bu \\ y &= Cx \end{cases} \quad (1)$$

Moment Matching techniques produce Reduced Order Models (ROM) whose transfer function are, up to a given order, equal to those of the HFM (1) at a given expansion point. This is done by the projection of the HFM into the subspace spanned by the columns of a matrix V, computed by the reduction algorithm, which results in the reduced system in (2).

$$\begin{cases} V^T E V\dot{x}_r &= V^T A V x_r + V^T B u \\ y_r &= C V x_r \end{cases} \quad (2)$$

To guarantee accuracy, complex expansion points are normally chosen. This will produce a complex projection matrix V and, therefore, a complex ROM. This poses problems to the physical interpretation of the reduced model. There are solutions to this problem that are, however, not very robust from a numerical point of view. In this paper, an alternative solution to produce a real subspace for the choice of complex expansions points is proposed.

II. Moment Matching

To produce ROMs whose transfer functions have the same moments of the HFM, the span of the projection matrix V must contain the subspace spanned by a matrix whose j-th column is given by (3). In this equation, s_0 is the complex expansion point. Therefore, V is, in general, complex.

$$V_j = \left[(A - s_0 E)^{-1} E\right]^{j-1} (A - s_0 E)^{-1} B \quad (3)$$

The direct computation of such matrix is normally done by the Arnoldi Iteration [4], used to build a unitary matrix whose columns span the desired subspace. At each iteration, the vector resulting from the matrix-vector product is orthogonalized to all the previously computed vectors. A real projection matrix can be obtained by placing the real and imaginary parts of the complex matrix side by side [5]. This, however, requires an additional rank-revealing orthogonalization process. Also, during the iterations, the vectors are being orthogonalized in respect to the complex vectors and not in respect to their real and imaginary parts, which results in partial loss of numerical robustness of the algorithm.

III. Modified Arnoldi Iteration

The proposed modified iteration, which uses an on the fly orthogonalization scheme, starts with a complex vector given by the first column of (3). The real and imaginary parts of this vector are separated and orthogonalized in relation to each other, producing an orthogonal V matrix of two columns. The next vector is obtained by the product of the matrix $(A - s_0E)^{-1}E$ to the last column of V. This will produce a new complex vector. First, the real part of this vector is orthogonalized to every column of V and added into a new column. Then, the same process is done to the imaginary part.

This should be repeated until the chosen number of moments are matched or deflation occurs for both real and imaginary parts. The resulting matrix is real, orthogonal and spans the desired subspace.

One must notice that the modified Arnoldi iteration and the classical one do not produce the same subspace for arbitrary matrices. This new approach only works because of the very special structure of the Krylov subspace when it is constructed for MOR applications.

IV. Application Example

As an example of application, a loop antenna, illustrated by Figure 1, obtained by the PEEC method [6] was reduced using both the classical technique and the proposed one.

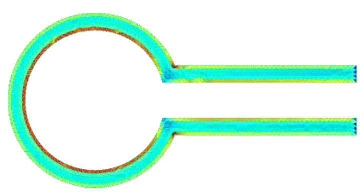

Fig. 1. Illustration of the loop antenna used as example.

This model has order 4650. Its output is the impedance of the system for different frequencies and its simulation for a thousand points takes around 26 minutes. Figure 2 shows the response of both reduced models. The time of the simulation for both ROM's are of the order of 20 ms.

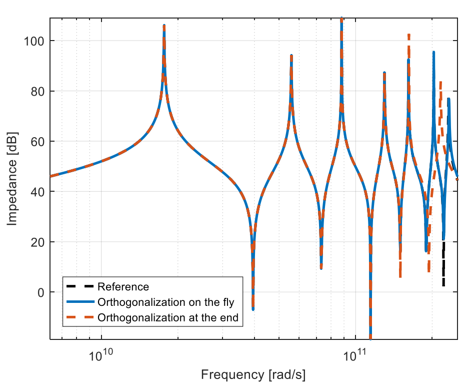

Fig. 2. Impedance of the system obtained using ROM's compared to the high-fidelity value.

Both ROM's used 1 GHz as the expansion point with 20 moments matched. It is then expected that the projection matrix contains 40 vectors, 2 for each moment matched. Table 1 contains some information about the reduction process for both methods. It is clear that the orthogonalization at the end of the strategy has produced subspaces with linearly dependent vectors. They are eliminated by the post processing step, thus spending computational time that does not translate into any additional accuracy of the ROM.

TABLE I
Results of the Reduction Process

Orthogonalization Technique	Average Reduction time [s]	Size of the Final Subspace	Discarded Vectors
at the End	8.75	35	5
on the Fly	8.46	40	0

The error for these two models in relation to the HFM can be seen in the Figure 3. The proposed method has lower error than the classical approach in the biggest part of the frequency range.

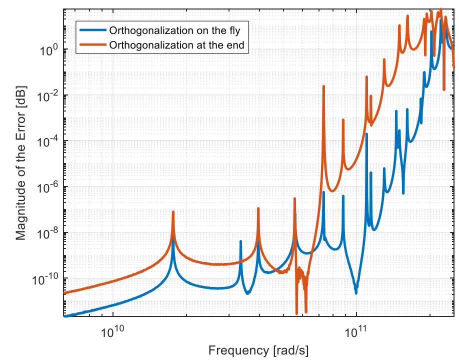

Fig. 3. Error obtained for the two different reduction approaches.

V. Conclusion

A new way of computing the Krylov subspace for projection based model order reduction was presented. An application example has shown that this new approach produces very encouraging results. The full paper will contain the mathematical demonstrations that the subspace spanned for the modified Arnoldi iteration is the same as the classical one in infinite precision arithmetic.

Acknowledgement

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